Design Results of RC Members Subjected to Bending, Shear, and Torsion Using ACI 318:08 and BS 8110:97 Building Codes

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Abstract: In this research, a comparative study was conducted on the amount of required reinforcement using American Concrete Institute (ACI) and British Standards Institution (BSI) building codes. The comparison included design cases of rectangular beam sections subjected to combined loads of bending, shear and torsion, and punching shear at slab–column connections. In addition, the study included comparison of the differences in the amount of reinforcement required owing to different codes' factors of safety for design loads. It was found that the BS code requires less reinforcement than the ACI code does for the same value of design load. However, when the load safety factors are included in calculating the design loads, the values of the resulting design loads become different for each code, and in this case, the ACI was found to require less reinforcement than the BS. The punching shear strength of flat slab–column connections calculated using the ACI code was found to be more than that calculated using the BS code for the same geometry, material, and loading conditions. The minimum area of flexural reinforcement required by ACI was found to be greater than by BS, while the opposite was found for the minimum area of shear reinforcement. In case both codes unify the load safety factors while keeping the other design equations as they are now, the BS code will have preference over the ACI code owing to lower reinforcement requirements, which leads to cheaper construction while maintaining safety. The study showed that both codes are good choices for design in Oman. Because SI units are becoming more and more enforced internationally, material that is available in Oman is conversant more toward SI units; to unify the knowledge of design among municipality and site engineers, it is recommended to use the BS code as a first choice until a national code is established. **DOI: 10.1061/(ASCE)SC.1943-5576.0000158.** © *2013 American Society of Civil Engineers.*

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Introduction

The structural design codes for RC, ACI 318:08 [American Concrete Institute (ACI) 2008] and BS 8110:97 [British Standards Institution (BSI) 1997], are based on the limit state design. However, these design codes differ on the design equations, especially for shear and torsion. They also differ on the factors of safety for material and loads. Because there is no national structural design code in Oman, a question about the most appropriate code in terms of safety, economy, and suitability to the environment in Oman is always asked. Knowledge of main features of and differences between the ACI 318 and BS 8110 codes is deemed a necessity. Although both ACI 318 and BS 8110 codes agree on the live load factor of safety to be 1.6, the factor of safety for the dead load in ACI is 1.2, whereas in BS it is 1.4, 17% greater. Subsequently, this results in a larger value of ultimate (design) load, which in turn affects the amount of reinforcement and concrete. The material strength reduction factor ϕ in the ACI 318 is 0.90 for flexure and 0.75 for shear and torsion, whereas in BS 8110, the material partial safety factor is 0.67 for flexure and 0.8 for shear and torsion. Further, unlike the ACI 318 code, where the reinforcement design strength is $A_s f_y$, the BS 8110 design strength is $0.95 A_s f_y$. The limit of maximum strain in concrete in ACI 318 is 0.003, whereas in BS 8110, the limit is 0.0035. Appendix I shows that the ACI 318 considers the material and geometry in defining the minimum area of longitudinal reinforcement, $A_{s,\min}$, whereas BS 8110 is based on geometry only. There is no differentiation between the sizes of b_w and b in considering the $A_{s,\min}$ in ACI 318, whereas BS 8110 has different values for $A_{s,\min}$ when $b_w/b < 0.4$ and $b_w/b \geq 0.4$. ACI 318 considers the effective depth d in calculating the geometry, whereas BS 8110 considers the total depth h. The ACI 318 equation

$$V_c = \left(0.16\sqrt{f_c'} + 17\rho\frac{V_u d}{M_u}\right)bd \; < 0.29\sqrt{f_c'}bd$$

(Section 11.2.2.1) assumes that shear strength of concrete is proportional to the square root of concrete cylinder compressive strength, whereas the BS 8110 equation

$$V_c = \left[\frac{0.79}{\gamma_m}\left(\frac{100A_s}{b_w d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\left(\frac{f_{cu}}{25}\right)^{1/3}\right]b_w d$$

(Section 3.4.5.4) assumes that the shear strength is proportional to the cubic root of cube concrete compressive strength. The maximum

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spacing between stirrups for torsional reinforcement in ACI 318 is the smaller of $p_h/8$ or 300 mm, whereas BS 8110 specifies the maximum spacing as the least of $0.8x_1y_1(0.95f_{yv})A_{sv,t}/T_u$, x_1, $y_1/2$, or 200 mm.

As per Jung and Kim (2008), the response of structural concrete to the actions of bending moment is quite well understood, and consequently, design procedures and provisions for bending moment are reasonably effective and consistent between different codes. Jung and Kim also stated: "Many of shear design code provisions are principally empirical, vary greatly from code to code, and do not provide uniform factors of safety against failure."

Sharma and Inniss (2006) found that the slab punching shear capacity v_c in ACI 318 is calculated from the concrete compressive strength as $0.33\sqrt{f_c'}$ without any consideration to the effect of longitudinal reinforcement, whereas in BS 8110

$$v_c = \frac{0.79}{\gamma_m}\sqrt[4]{\frac{400}{d}}\sqrt[3]{\frac{100A_s}{b_w d}}\sqrt[3]{\frac{f_{cu}}{25}}$$

which takes account of the longitudinal reinforcement in addition to concrete strength.

Subramanian (2005) pointed out that in BS 8110, the critical section for checking the punching shear is $1.5d$ from edge of load point, whereas in ACI 318, the critical section for checking punching shear is $0.5d$ from edge of load point.

Bari (2000) reviewed the shear strength of slab–column connections and concluded that the BS code predicts smaller shear strength than ACI for values of ρ less than 1.2% and larger strength for values of ρ greater than 1.2%. However, this limit may vary for different column shapes, concrete strengths, and effective depths. Bari also concluded that with a ratio of column side length of 2.5 to 5, the BS code predicts greater strength than the ACI code, whereas for ratio ranges between 1 and 2.5, ACI predicts more shear strength than BS.

Ngo (2001) stated: "Depending on method used, the critical section for checking punching shear in slabs is usually situated between 0.5 to 2 times the effective depth from edge of load or the reaction." He concluded that the punching shear strength values that are specified in different codes vary with concrete compressive strength f_c' and are usually expressed in terms of $(f_c')^n$. In ACI 318, the punching shear strength is expressed as proportional to $\sqrt{f_c'}$, whereas in BS 8110, punching shear strength is assumed to be proportional to $\sqrt[3]{f_{cu}}$.

Chiu et al. (2007) carried out a parametric study based on ACI 318:05 (ACI 2005) and found that torsional strength decreases as the aspect ratio (longer dimension/shorter dimension) of specimen increases.

Bernardo and Lopes (2009) analyzed several codes of practice regarding torsion and concluded that the ACI code has clauses that impose maximum and minimum amounts of torque reinforcement (for both transverse and longitudinal bars). The equations for minimum amount of reinforcement are, however, mainly empirical and sometimes lead to questionable solutions, namely negative minimum longitudinal reinforcement or disproportional longitudinal reinforcement and stirrups.

According to Ameli and Ronagh (2007), the area used in shear flow calculation is determined differently in different codes, which results in different torsional shear strengths. Taking the centers of longitudinal bars or center-to-center of stirrups for the calculation of this area will result in different sizes of area.

Alnuaimi and Bhatt (2006), reported that "most researchers believe that the shear stress owing to direct shear is resisted by the whole width of cross section while the torsional shear stress is resisted by the outer skin of concrete section. They differ, however, on the thickness of outer skin."

Based on the literature, it is clear that some research works have been carried out on the comparison between ACI and BS codes. However, the comparisons were limited to few parameters and d not touch the effects of these differences on the amount of rein forcement. No study was found in the literature on the preference o design codes for structural design in Oman or the rest of the Gul states. In this research, an intensive comparison work was carrie out to find out the effects of design results on the amount of rein forcement using ACI and BS codes. Effects of different parameter were studied including M_u/V_u ratio, load safety factors, require length for transverse reinforcement, minimum flexural and shear re inforcement, etc. A recommendation on a preferred code is presente

Design Equations

Bending

The design procedures in ACI 318:08 and BS 8110:97 are based o the simplified rectangular stress block as given in ACI 318:08–10. and BS 8110:97–3.4.4, respectively. The area of required flexur reinforcement in ACI 318:08–10.3.4 is given as

$$A_s = \frac{M_u}{\phi f_y\left(d - \frac{a}{2}\right)} \qquad (1$$

where

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85f_c'\phi b}}$$

In BS 8110:97–3.4.4.4, the area of required reinforcement given by

$$A_s = \frac{M_u}{0.95f_y z} \qquad ($$

where

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d$$

and $K = M_u/f_{cu}bd^2$.

Shear

The concrete shear strength, v_c, in a beam can be calculated from AC 318:08–11.2.2.1 as the resulting smaller value of

$$v_c = \text{min of} \begin{bmatrix} \left(0.16\sqrt{f_c'} + 17\rho\frac{V_u d}{M_u}\right) \\ 0.29\sqrt{f_c'} \end{bmatrix} \qquad ($$

where $f_c' \le 70$ N/mm²; and $V_u d/M_u \le 1$.

According to Table 3.8 of BS 8110:97–3.4.5.4, the concre shear strength, v_c, is calculated as

$$v_c = \frac{0.79}{\gamma_m}\left(\frac{100A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\left(\frac{f_{cu}}{25}\right)^{1/3} \qquad ($$

with the following limitation: $\gamma_m = 1.25$, $0.15 \le 100A_s/bd \le$ $(400/d) \ge 1$ and $f_{cu} \le 40$ N/mm².

The required shear reinforcement, A_{sv}/S, for different values shear force is calculated based on ACI 318:08–11.4 as

$$\frac{A_{sv}}{S} = \begin{bmatrix} 0 & \text{for } V_u < \dfrac{\phi V_c}{2} \\ \dfrac{0.062\sqrt{f_c'}b}{f_y} > \dfrac{0.35bS}{f_y} & \text{for } \dfrac{\phi V_c}{2} \leq V_u \leq \phi V_c \\ \dfrac{V_u - \phi V_c}{\phi d f_y} & \text{for } V_u \geq \phi V_c \end{bmatrix} \tag{5}$$

The minimum transverse reinforcement A_{sv}/S_v for beam is calculated based on Table 3.7 of BS 8110:97–3.4.5.3 and BS 8110:97–3.4.5.2 as

$$\frac{A_{sv}}{S_v} = \begin{bmatrix} \dfrac{0.4b}{0.95 f_{yv}} & \text{for } v \leq (v_c + 0.4) \\ \dfrac{v - v_c b}{0.95 f_{yv}} & \text{for } (v_c + 0.4) < v \leq v_{max} \\ \text{resize section} & \text{for } v > v_{max} \end{bmatrix} \tag{6}$$

The punching shear strength in flat slab is defined by ACI 318:08–11.11.2.1 as the smallest of

$$V_c = \min \text{ of} \left\{ \begin{array}{c} \left[0.17\left(1 + \dfrac{2}{\beta}\right)\sqrt{f_c'}\right]bd \\ \left[0.83\left(\dfrac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c'}\right]bd \\ \left(0.33\sqrt{f_c'}\right)bd \end{array} \right\} \tag{7}$$

The maximum shear stress, $v_{\max}$, is defined in BS 8110:97–3.4.5.2 as

$$v_{\max} = 0.8\sqrt{f_{cu}} \leq 5 \text{ N/mm}^2 \tag{8}$$

Torsion

The design provision for torsional cracking strength of RC solid beam in ACI 318:08–11.5.1 is specified as

$$T_{cr} = \frac{\sqrt{f_c'}}{3}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{9}$$

If $T_u < \phi T_{cr}/4$, no torsional reinforcement is needed.

The torsional strength of a member is given by ACI 318:08–11.5.3.5 as

$$T_n = \frac{2A_t A_o f_{yv}}{s}\cot\theta \tag{10}$$

where $A_o = 0.85A_{oh}$ and $\theta = 45°$ for RC member.

In BS 8110–2:85 (BSI 1985)–2.4.4.1, the torsional shear stress, v_t, for rectangular beam is computed as

$$v_t = \frac{2T_u}{h_{\min}^2\left(h_{\max} - \dfrac{h_{\min}}{3}\right)} \tag{11}$$

The minimum torsion stress, $v_{t,\min}$, below which torsion in the section can be ignored based on BS 8110–2:85–2.4.6, is given for different grades of concrete as

$$v_{t,\min} = 0.067\sqrt{f_{cu}} \leq 0.4 \text{ N/mm}^2 \tag{12}$$

If $v_t > v_{t,\min}$ then torsional resistance is to be provided by closed stirrups and longitudinal bars.

Based on ACI 318:08–11.5.3.7, the required longitudinal reinforcement is calculated as

$$A_l = \frac{A_t}{s}P_h\left(\frac{f_{yv}}{f_{yl}}\right)\cot^2\theta \tag{13}$$

ACI 318:08–11.5.5.3 specifies the minimum longitudinal torsional reinforcement as

$$A_{l,\min} = \frac{0.42\sqrt{f_c'}\,A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right)P_h\frac{f_{yv}}{f_{yl}} \tag{14}$$

where $A_t/s \leq 0.175b_w/f_{yv}$.

In addition, ACI 318:08–11.5.6.2 restricts the maximum spacing between bars of the longitudinal reinforcement required for torsion to 300 mm. The longitudinal bars shall be inside the closed stirrups and at least one bar is required in each corner. Longitudinal bar diameter shall not be less than 10 mm.

The required longitudinal reinforcement due to torsion is given in BS 8110–2:85–2.4.8 as

$$A_l = \frac{A_{sv,t} f_{yv}(x_1 + y_1)}{s_v f_y} \tag{15}$$

BS 8110–2:85–2.4.9 states that the longitudinal torsion reinforcement shall be distributed evenly around the perimeter of stirrups. The clear distance between these bars should not exceed 300 mm. Longitudinal bar diameter shall not be less than 12 mm.

The required stirrups, (by assuming $\theta = 45°$) is given by ACI 318:08–11.5.3.5 as

$$\frac{A_t}{s} = \frac{T_u}{1.7\phi A_{oh} f_{yv}} \tag{16}$$

For pure torsion, the minimum amount of closed stirrup is specified by ACI 318:08–11.5.5.2 as the greater result of the following

$$A_{t,\min} = \text{larger of} \begin{bmatrix} \dfrac{2A_{t,\min}}{s} = 0.062\sqrt{f_c'}\dfrac{b_w}{f_{yv}} \\ \dfrac{2A_{t,\min}}{s} \geq 0.35\dfrac{b_w}{f_{yv}} \end{bmatrix} \tag{17}$$

ACI 318:08–11.5.6.1 specifies the maximum spacing of stirrups as the smaller of $p_h/8$ or 300 mm.

The shear reinforcement made of closed stirrups is calculated based on BS 8110–2:85–2.4.7 as

$$\frac{A_{sv,t}}{S_v} = \frac{T_u}{0.8x_1 y_1(0.95 f_{yv})} \tag{18}$$

S_v should not exceed the least of x_1, $y_1/2$ or 200 mm.

To prevent crushing of surface concrete of solid section, ACI 318:08–11.5.3.1 restricts the cross-sectional dimension as

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) \tag{19}$$

where $V_c = 0.17\sqrt{f_c' bd}$.

For section dimensions check in BS 8110–2:85–2.4.5, the computed torsional shear stress, v_t, should not exceed the following limit for sections with larger center-to-center dimensions of closed link less than 550 mm

$$v_t \leq v_{tu} \frac{y_1}{550} \tag{20}$$

In no case should the sum of the shear stresses resulting from shear force and torsion ($v_{tu} = v_s + v_t$) exceed the maximum shear stress specified by Eq. (8). If a combination of applied shear stress v_s and torsional stress v_t exceeds this limit, the section should be resized.

Design Results and Discussions

Here, design results of rectangular beams with different load combinations and span/depth ratios are presented. The ACI 318:08 and BS 8110:97 codes were used in the design, and results were judged based on the amount of longitudinal and transverse reinforcement requirements. The characteristic cube compressive strength of concrete was 30 N/mm^2 and the cylinder compressive strength was 24 N/mm^2 with concrete density of 24 kN/m^3, and the characteristic yield strength of the longitudinal and transverse reinforcement was 460 N/mm^2.

Design for Combined Bending Moment and Shear Force Using ACI 318:08 and BS 8110:97

Tables 1–3 and Table 4 show the design results of three groups of simply supported beams and one group of two-span continuous beams, respectively. In beam numbering, the first letter denotes the type of member considered, e.g., B means beam; the second letter denotes the variable, e.g., R means span/depth ratio; the third letter denotes the type of loading, e.g., W means uniformly distributed load. The first numeral represents the value of R and the second numeral represents the value of W. The beam cross-sectional dimension was selected to be 350 × 700 mm with effective depth of 625 mm. Different ultimate design uniformly distributed load values were used as shown in the caption of each table. It was assumed that 50% of bottom longitudinal bars are curtailed at 0.1L from support, and transverse stirrups were used for shear reinforcement, i.e., no bent-up bars considered to resist shear. The span/depth ratio was varied among each group resulting in different M_u/V_u ratios. The design was carried out using the ACI and BS codes for the same ultimate design load values.

It is clear that the two codes gave almost the same results fo bending reinforcement, with minimal effect of M_u/V_u ratio changes. giving a maximum difference of 2.6% in the case of single-spar simply supported beams and 0.84% in the case of the continuou beams. However, the results differ largely on the shear reinforcemen with the change of M_u/V_u ratio using ACI and BS codes, as can b

Table 1. Simply Supported Beams with Ultimate Design UDL of 75 kN/m

Beam number	Span (m)	L/d Ratio	M_u at midspan (kNm)	V_u at d from support (kN)	M_u/V_u (kNm/kN)	A_s(mm^2)		Difference in A_s (%)	A_{sv}/s at support (mm^2/mm)		Difference in A_{sv}/s (%)	Length of region for shear reinforcement (m)		Difference in length of shear reinforcemer (%)
						ACI	BS		ACI	BS		ACI	BS	
BR11.2W75	7	11.2	459	216	2.13	1,975	1,962	0.7	0.35	0.37	5.7	1.75	0.83	110.8
BR12W75	7.5	12	527	234	2.25	2,312	2,326	0.6	0.43	0.42	2.4	1.90	1.00	90.0
BR12.8W75	8	12.8	600	253	2.37	2,692	2,754	2.3	0.50	0.47	6.4	2.25	1.17	92.3

Table 2. Simply Supported Beam with Ultimate Design UDL of 100 kN/m

Beam number	Span (m)	L/d Ratio	M_u at midspan (kNm)	V_u at d from support (kN)	M_u/V_u (kNm/kN)	A_s (mm^2)		Difference in A_s (%)	A_{sv}/s at support (mm^2/mm)		Difference in A_{sv}/s (%)	Length of region for shear reinforcement (m)		Difference in length of shear reinforceme (%)
						ACI	BS		ACI	BS		ACI	BS	
BR8.8W100	5.5	8.8	378	213	1.78	1,591	1,571	1.3	0.35	0.40	14.3	1.40	0.83	68.7
BR9.6W100	6	9.6	450	238	1.89	1,931	1,916	0.8	0.46	0.46	0.0	1.65	1.01	63.4
BR11.2W100	7	11.2	613	288	2.13	2,762	2,835	2.6	0.67	0.59	13.6	2.15	1.36	58.1

Table 3. Simply Supported Beam with Ultimate Design UDL of 125 kN/m

Beam number	Span (m)	L/d Ratio	M_u at midspan (kNm)	V_u at d from support (kN)	M_u/V_u (kNm/kN)	A_s (mm^2)		Difference in A_s (%)	A_{sv}/s at support (mm^2/mm)		Difference in A_{sv}/s (%)	Length of region for shear reinforcement (m)		Differenc in length of shear reinforcem (%)
						ACI	BS		ACI	BS		ACI	BS	
BR8W125	5	8	391	234	1.67	1,652	1,624	1.7	0.45	0.47	4.4	1.4	0.95	47.4
BR9.6W125	6	9.6	563	297	1.89	2,497	2,532	1.4	0.72	0.64	12.5	1.90	1.30	46.2
BR10.4W125	6.5	10.4	660	328	2.01	3,078	3,089	0.4	0.85	0.72	18.1	2.2	1.5	46.7

seen for a typical beam in Fig. 1. The differences become pronounced with the increase of M_u/V_u ratio, leading to continually diverging curves. In most cases, it was found that the BS requires less transverse reinforcement than the ACI. For the given geometry and loads, the differences reached up to 18.1% in the case of simply supported beams and 31.4% in the case of continuous beams. Further, the length from the face of support to the point beyond which only minimum shear reinforcement is required was also investigated and is presented in the penultimate columns of Tables 1–4. It was found that the length that needs shear reinforcement required by BS is less than that required by ACI. The differences become more pronounced with increase of M_u/V_u ratio. For the given geometry and loads, the differences reached up to 110.8% in the case of simply supported beams and 153.3% in the case of continuous beams. This indicates that with the increase of loads, the BS code becomes more economical on the transverse reinforcement.

Table 5 shows comparison between the ACI and BS results on the shear strength capacity of concrete using

$$v_c = \left(0.16\sqrt{f_c'} + 17\rho\frac{V_u d}{M_u}\right) \leq 0.29\sqrt{f_c'}$$

as extracted from Eq. (3) in the case of ACI code and Eq. (4) in the case of BS code for different values of ρ ranging from 0.2 to 2.0%. In the ACI, the values of $V_u d/M_u$ were varied from 0 to 1.0 and in the case of BS the value of $400/d$ was taken as constant equal to 1. It is obvious that the above equations lead to highly different results. Initially, when reinforcement ratio, ρ, is 0.2%, v_c of BS is less by about 60% than that of ACI for all values of $V_u d/M_u$. As the $V_u d/M_u$ and/or ρ increases, the concrete shear capacity increases. Fig. 2 shows that the nonlinear curve resulting from the BS equation crosses the linearly diverging curves made by the ACI equation for variable values of $V_u d/M_u$ at different points. The first crossing point occurred at $\rho = 0.8\%$ with $V_u d/M_u = 0$. The succeeding crosses occurred sequentially with the increased $V_u d/M_u$ curves. It is also clear that the BS rate of increase in shear capacity is more rapid than that of the ACI. Appendix II shows formulation of both ACI and BS code equations for required shear reinforcement to produce two similar equations with differences in the empirical values. It can be seen that even in the case when v_c in ACI is equal to v_c in BS, as shown in the crosses of Fig. 2, the values of spacing between stirrups, S, as shown by the resulting equations in Appendix II, will be different and the ACI code requires approximately 26% more shear reinforcement than the BS code. This difference is attributed to differences in material safety factors.

Design for Torsion Using ACI 318:08 and BS 8110:97

Here, 500 × 700-mm beams with effective depth of 625 mm were subjected to pure twisting moment. Table 6 shows the design results of fixed beams subjected to ultimate design torsion using ACI and BS codes. In beam numbering, the first letter denotes the type of member, e.g., B means beam; the second letter denotes the variable, e.g., L means span; and the numeral gives the value of L. It is clear that the required longitudinal reinforcement, by ACI, is 19.2% larger than that required by BS for most of the beams (i.e., BL6, BL8, and BL10). Because BL4 needs minimum steel in the ACI approach, this

Table 4. Two Span Continuous Beam with Ultimate Design UDL of 60 kN/m

Beam number	Span (m)	L/d Ratio	M_u at midspan (kNm)	V_u at d from support (kN)	M_u/V_u (kNm/kN)	A_s (mm^2)		Difference in A_s (%)	A_{sv}/s at support (mm^2/mm)		Difference in A_{sv}/s (%)	Length of region for shear reinforcement (m)		Difference in length of shear reinforcement (%)
						ACI	BS		ACI	BS		ACI	BS	
BR12W60	7.5	12	338	221	1.53	1,409	1,375	0.84	0.36	0.35	2.9	1.90	0.75	153.3
BR13.6W60	8.5	13.6	434	256	1.70	1,855	1,835	0.67	0.52	0.43	20.9	2.50	1.12	123.2
BR15.2W60	9.5	15.2	542	290	1.87	2,389	2,410	0.56	0.67	0.51	31.4	3.00	1.48	102.7

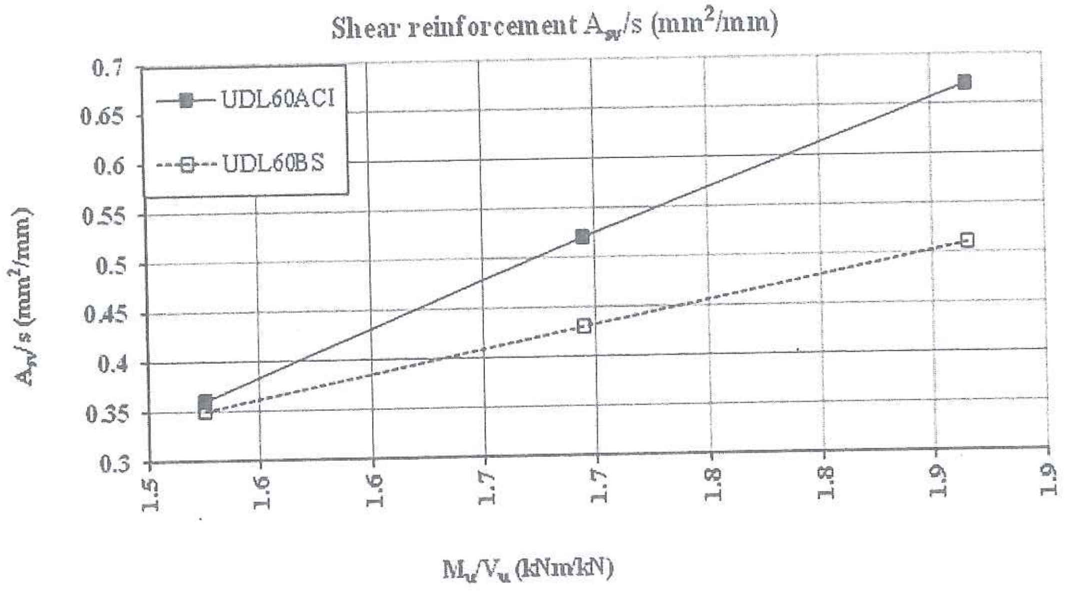

Fig. 1. Shear reinforcement versus M_u/V_u using ACI and BS codes (continuous beams)

beam was not considered in the comparison. The transverse reinforcement required by ACI is approximately 19% larger than that required by BS. This shows that BS is more economical than ACI in the case of design for torsion in RC rectangular solid beams.

Appendix I shows a comparison between the ACI and BS torsion equations that lead to required transverse and longitudinal reinforcement. It is clear that the area of the shear flow A_{oh} is taken as $0.85x_1y_1$ in ACI, whereas in BS, it is taken as $0.8x_1y_1$. Further, owing to differences in material safety factors, ACI required about 19% more transverse torsional reinforcement than BS. Regarding longitudinal reinforcement, the derived Eqs. (2) and (4) in Appendix I look identical in both codes, but since longitudinal reinforcement is dependent on the amount of transverse reinforcement, the same difference of 19% that was found above for transverse reinforcement is carried to longitudinal reinforcement.

Design for Combined Bending Moment, Shear Force, and Twisting Moment Using ACI 318:08 and BS 8110:97

Tables 7 and 8 show the design results of longitudinal reinforcement for two groups of fixed end beams with different uniformly distributed load values and torsional moment of 12.5 kNm/m as shown in the caption of each table. The beam size considered was 400 × 700 mm with effective depth of 625 mm, and design results were calculated near the support. In beam numbering, the first letter denotes the type of member considered, e.g., B means beam; the second letter denotes the variable, e.g., R means span/depth ratio; the third letter denotes the type of loading, e.g., W means uniformly distributed load. The first numeral represents the value of R and the second numeral represents the value of W. It is clear that the required top reinforcement for ACI is larger than that for BS, with a maximum difference of 8.4% for the given loads and beam geometry. The bottom and face reinforcement required by ACI is larger by about 19.3% than that required by BS. No major changes were found in the longitudinal reinforcement due to the increase of L/d or M_u/T_u ratios. However, the results differ largely on the transverse reinforcement with the change of V_u/T_u ratio using ACI and BS codes, as can be seen in Fig. 3. It can be seen that the ACI curve is linear whereas the BS curve is nonlinear. The differences become pronounced with increase of V_u/T_u ratio, leading to continually diverging curves. In most cases, it was found that BS requires less transverse reinforcement than ACI. For the given geometry and loads, the difference reached up to 19.3%.

Table 5. Concrete Shear Stress Capacity, v_c

	V_ud/M_u					Concrete shear stress, v_c (N/mm^2) (BS 8110) with $400/d = 1$
	0	0.25	0.5	0.75	1.00	
ρ (%)	Concrete shear stress, v_c (N/mm^2) (ACI 318)					
0.20	0.784	0.792	0.801	0.809	0.818	0.493
0.40	0.784	0.801	0.818	0.835	0.852	0.620
0.60	0.784	0.809	0.835	0.860	0.886	0.709
0.80	0.784	0.818	0.852	0.886	0.920	0.779
1.00	0.784	0.826	0.869	0.911	0.954	0.839
1.20	0.784	0.835	0.886	0.937	0.988	0.891
1.40	0.784	0.843	0.903	0.962	1.022	0.938
1.60	0.784	0.852	0.920	0.988	1.056	0.980
1.80	0.784	0.860	0.937	1.013	1.090	1.019
2.00	0.784	0.869	0.954	1.039	1.124	1.055

Impact of Load Safety Factors on Design Load Using ACI 318:08 and BS 8110:97

In this section, simply supported RC beams of 200- × 700-mm cross section, 625-mm effective depth, and 6-m effective span with uniformly distributed live and dead loads were designed using the ACI and BS codes. The live load was kept constant at 5 kN/m for all beams while the dead load values were varied from 20 to 40 kN/m. The live load was kept constant because the factor of safety for the live load is the same in both ACI and BS codes, i.e., 1.6. It was assumed that 50% of bottom bars are curtailed at $0.1L$ from the center of support. Table 9 shows the effects of the ACI and BS code factors of safety on required reinforcement. In the beam numbering, the first letter denotes the type of member, e.g., B means a beam; the second letter denotes the variable, e.g., R is the ratio of dead load to live load (DL/LL); and the numeral gives the value of R. It is clear that because of the different values of dead load factors of safety, 1.2 in ACI and 1.4 in BS, the differences in design bending moments and shear forces between the results from ACI and BS are linearly increasing with the increase of the dead load. For the given service loads, the factored (ultimate)

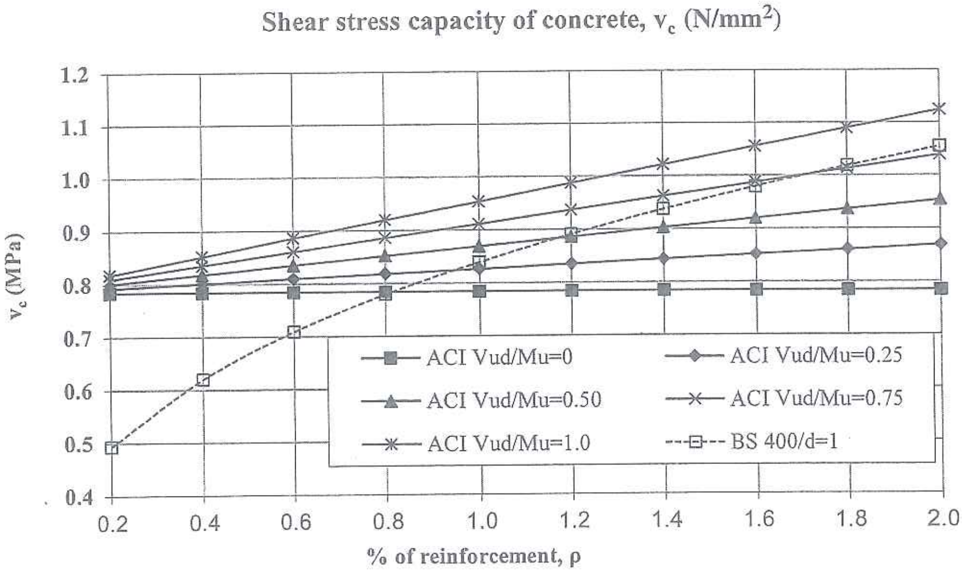

Fig. 2. Concrete shear stress v_c versus ρ using ACI and BS codes

design load using BS was larger than that for ACI, having a maximum difference of 14.3%. As a result, to these load differences, the required longitudinal and transverse reinforcements are differing with maximum of 16.5% for bending and 60.0% for shear reinforcements, respectively. The results for the flexural reinforcement indicates slight diversion due to the effect of increasing dead load, while the required shear reinforcement shows convergence on the required transverse reinforcement with the increase of the DL/LL ratio. Beams BR4 and BR5 required minimum stirrups in ACI and hence are not considered in the discussion. It is interesting to notice that, as seen in Tables 1–4, for the ultimate design loading, the difference in flexural reinforcement using ACI and BS is negligible. For service loading, however, shown in Table 9, the required flexural reinforcement for BS was larger than for ACI, with differences varying from 9.9 to 16.5%. This difference is attributed to the different load safety factors that are used in ACI and BS for dead and live load combinations. Similarly, as seen in Tables 1–4, for the ultimate design loading, the shear reinforcement required by BS is less compared with ACI, whereas for service loading (Table 9), the result is reversed. This reversal of result is also attributed to the different load safety factors used in ACI and BS codes for the dead load.

Table 6. Fixed Beams with Ultimate Design Torsion

Beam number	Span (m)	T_u (kN.m)	A_l (mm^2) ACI	A_l (mm^2) BS	Difference in A_l (%)	A_t/s (mm^2/mm) ACI	A_t/s (mm^2/mm) BS	Difference in A_t/s (%)
BL4	4	50	min	583	—	0.68	0.57	19.3
BL6	6	75	1,043	875	19.2	1.02	0.86	18.6
BL8	8	100	1,391	1,167	19.2	1.36	1.14	19.3
BL10	10	125	1,738	1,458	19.2	1.7	1.43	18.9

Punching Shear Strength (at Slab–Column Connection) Using ACI 318:08 and BS 8110:97

Here, a parametric study of punching shear capacity at slab–column connection, using ACI and BS codes, was carried out with different column aspect ratios, percentages of flexural reinforcement, and slab thicknesses. The characteristic cube and cylindrical compressive strengths were taken as 35 and 28 N/mm^2, respectively, and the characteristic yield strength of reinforcement was taken as 460 N/mm^2.

Table 7. Fixed End Beams with Ultimate Design UDL of 100 kN/m and Torsion of 12.5 kNm/m

Beam number	Span (m)	L/d ratio	M_u (kNm)	V_u at d (kN)	T_u at d (kNm)	M_u/V_u	V_u/T_u	Top steel (mm^2) ACI	Top steel (mm^2) BS	Difference in top bars (%)	Bottom steel (mm^2) ACI	Bottom steel (mm^2) BS	Face bars (mm^2) ACI	Face bars (mm^2) BS	Difference in top/face bars (%)
BR8W100	5	8	208	188	66	3.17	2.86	1,200	1,107	8.4	365	306	365	306	19.3
BR9.6W100	6	9.6	300	238	67	4.47	3.53	1,598	1,498	6.7	371	311	371	311	19.3
BR11.2W100	7	11.2	408	288	68	5.98	4.21	2,084	1,986	4.9	376	315	376	315	19.4

Table 8. Fixed End Beams with Ultimate Design UDL of 125kN/m and Torsion of 12.5 kNm/m

Beam number	Span (m)	L/d ratio	M_u (kNm)	V_u at d (kN)	T_u at d (kNm)	M_u/V_u	V_u/T_u	Top steel (mm^2) ACI	Top steel (mm^2) BS	Difference in top bars (%)	Bottom steel (mm^2) ACI	Bottom steel (mm^2) BS	Face bars (mm^2) ACI	Face bars (mm^2) BS	Difference in top/face bars (%)
BR8W125	5	8	260	234	66	3.97	3.57	1,420	1,323	7.3	365	306	365	306	19.3
BR9.6W125	6	9.6	375	297	67	5.58	4.42	1,929	1,830	5.4	371	311	371	311	19.3
BR11.2W125	7	11.2	510	359	68	7.47	5.26	2,563	2,483	3.2	376	315	376	315	19.4

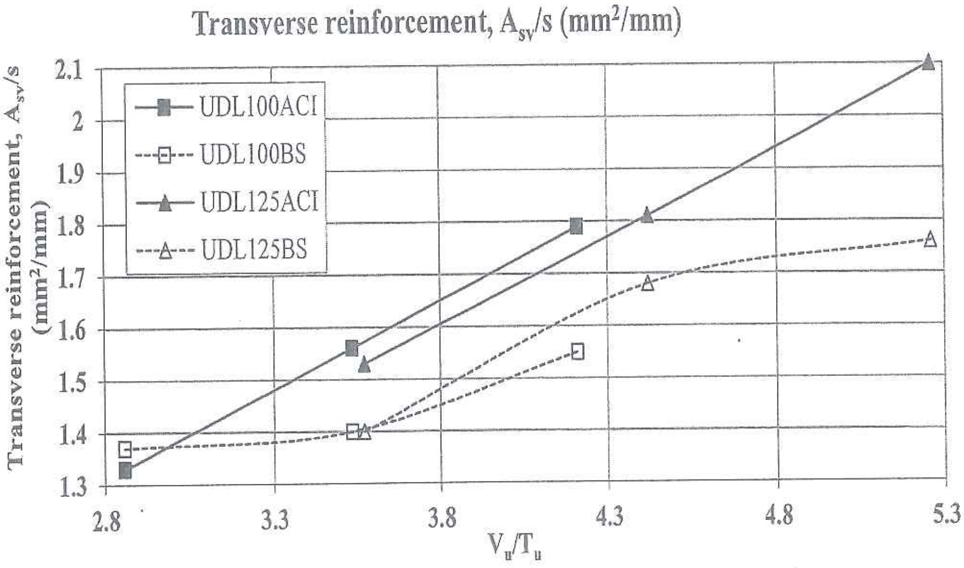

Fig. 3. Transverse reinforcement versus V_u/T_u (for UDL = 100 and 125 kN/m)

From Eqs. (4) and (7), it can be seen that unlike BS 8110, ACI 318 does not consider dowel action of flexural reinforcement in the calculation of shear capacity. Fig. 4 shows punching shear strength of a 250-mm-thick slab with effective depth of 220 mm at interior column having $\rho = 1.0\%$ with column sizes of 300×300, 300×600, 300×900, and 300×1200 mm, resulting in different aspect ratios using ACI and BS codes. The concrete cube compressive strength, f_{cu}, was 35 $\mathrm{N/mm^2}$, with concrete cylinder compressive strength as $0.8 f_{cu}$ and steel yield strength of $460\ \mathrm{N/mm^2}$. It can be seen that punching shear strength for ACI is larger than for BS for all aspect ratios. This means that for the same ultimate design punching shear force, ACI requires less slab thickness than BS. The largest difference is 36.8% when column aspect ratio is 2. It can also be seen that, in BS code, punching shear strength increases linearly as column aspect ratio increases, whereas in ACI code, the curve is nonlinear, having a larger rate of increase of punching shear strength between column aspect ratio of 1 and 2 than between 2 and 4. Fig. 5 shows punching shear strength of 250-mm-thick slab at interior column of size 300×300 mm with different percentages of flexural reinforcement ranging between 0.15 and 3%, using ACI and BS codes. The material strengths and effective depths of slab were the same as in the previous slab. It can be seen that, in ACI code, the punching shear strength is constant at 773 kN without any effect of the percentage of flexural reinforcement, whereas in BS code, punching shear strength increases with increase of reinforcement. The BS curve is nonlinear and it crosses the ACI horizontal line at $\rho = 1.7\%$. This shows that with the given data, until $\rho = 1.7\%$, ACI leads to larger punching shear strength than BS, with the largest difference of 131.4% when $\rho = 0.15\%$.

Fig. 6 shows punching shear strength of slab at interior column of size 300×300 mm with area of flexural reinforcement $A_s = 2,050\ \mathrm{mm^2}$ with varying depth. The material strengths were the same as in the previous slab. The effective depth of each slab is 35 mm less than the overall thickness. It can be seen that ACI estimates more punching shear strength than BS. The differences, with the given data, ranged between 14.8 and 23.3%. Both ACI and BS curves vary linearly with increase of depth; however, the rate of increase in the ACI results is more than in BS, leading to diverging curves. This shows that the difference in V_c, using both codes, increases as depth increases.

Comparison for Minimum Area of Flexural Reinforcement Using ACI 318:08 and BS 8110:97

The equations of minimum required flexural reinforcement based on the ACI and BS codes are shown in Appendix I. Fig. 7 was developed based on those equations for different values of f_c', which is taken as $0.8 f_{cu}$. The beam cross-sectional dimension is 350×700 mm with effective depth of 625 mm. The yield strength of reinforcement was taken as $460\ \mathrm{N/mm^2}$. It can be seen that the minimum area of flexural reinforcement required by ACI is much larger than that required by BS. The BS curve is constant with all grades of concrete,

Table 9. Parametric Study to Compare Steel Required for Bending and Shear with DL + LL Combination Using ACI and BS

Beam number	Ratio DL:LL	Service UDL (kN/m) Dead	Service UDL (kN/m) Live	Ultimate design UDL (w_u) (kN/m) ACI (1.2D + 1.6L)	Ultimate design UDL (w_u) (kN/m) BS (1.4D + 1.6L)	Difference in w_u (%)	Ultimate design moment at midspan M_u (kNm) ACI	Ultimate design moment at midspan M_u (kNm) BS	Ultimate design shear at d V_u (kN) ACI	Ultimate design shear at d V_u (kN) BS	Flexural reinforcement A_s ($\mathrm{mm^2}$) ACI	Flexural reinforcement A_s ($\mathrm{mm^2}$) BS	Difference in A_s (%)	Shear reinforcement A_{sv}/s ($\mathrm{mm^2/mm}$) ACI	Shear reinforcement A_{sv}/s ($\mathrm{mm^2/mm}$) BS	Difference in A_{sv}/s (%)
BR4	4	20	5	32	36	12.5	144	162	76	86	588	646	9.9	min	0.18	—
BR5	5	25	5	38	43	13.2	171	194	90	102	706	789	11.8	min	0.18	—
BR6	6.5	33	5	47	53.5	13.8	212	241	112	127	891	1,014	13.8	0.15	0.24	60.0
BR7	7	35	5	50	57	14.0	225	257	119	135	951	1,094	15.0	0.18	0.26	44.4
BR8	8	40	5	56	64	14.3	252	288	133	152	1,079	1,257	16.5	0.24	0.31	29.2

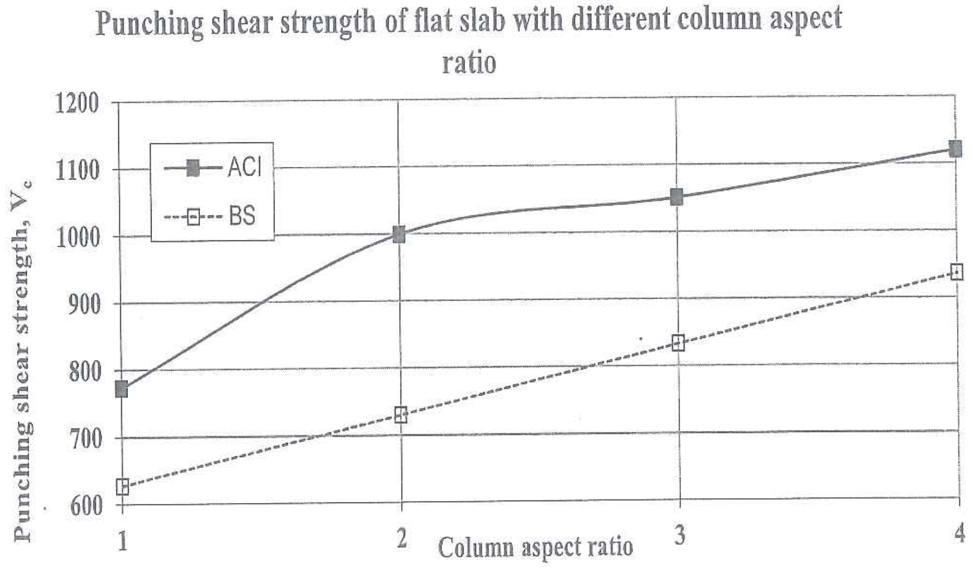

Fig. 4. Punching shear strength versus column aspect ratio using ACI and BS codes

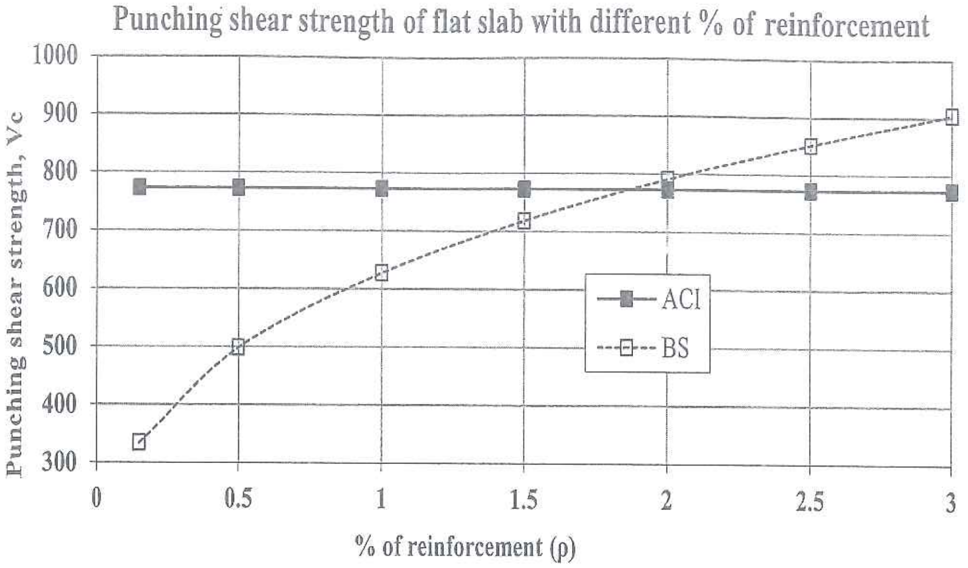

Fig. 5. Punching shear strength versus percentage of ρ using ACI and BS codes

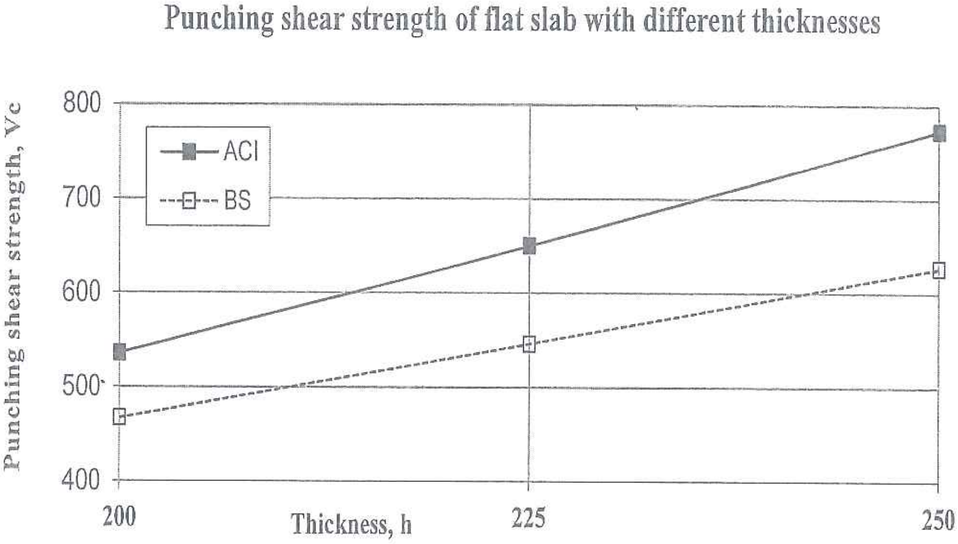

Fig. 6. Punching shear strength versus slab thickness using ACI and BS codes

while the ACI curve changes from being constant for concrete grades of 30–40 N/mm^2 to nonlinear for concrete grades larger than 40 N/mm^2. The difference is constant at the value of 106% for the concrete strengths of 30–40 N/mm^2, then it increases with the increase in the concrete strength. The maximum difference for the given beam geometry and concrete strengths was 133.5%.

Comparison for Minimum Area of Shear Reinforcement Using ACI 318:08 and BS 8110:97

Fig. 8 was developed based on Eqs. (5) and (6) for different values of f_c', which is taken as $0.8f_{cu}$. The beam cross-sectional dimension is 350×700 mm with effective depth of 625 mm. The yield strength of reinforcement was taken as 460 N/mm^2. It can be seen that the minimum area of shear reinforcement required by BS is larger than required by ACI. The BS curve is constant for all grades of concrete, whereas the ACI curve is constant for concrete grades of 30–40 N/mm^2 then reduces linearly for grades larger than 40 N/mm^2. The difference is constant at the value of 18.5% for the concrete strengths between 30 and 40 N/mm^2 then decreases with the increase in the concrete strength. The minimum difference for the given beam geometry and concrete strengths was 6.7% at concrete grade of 50 N/mm^2.

Concluding Remarks and Recommendations

In this research, design results of rectangular RC beams subjected to bending, shear and torsion, and punching shear at the slab–column connection, using ACI 318:08 and BS 8110:97, were compared. Conclusions can be drawn as follows.

Design for Combined Bending Moment, Twisting Moment, and Shear Force

- The required flexural reinforcements for the same design bending moment, using ACI and BS codes, are almost the same regardless of M_u/V_u ratio.
- In most cases, the required shear reinforcement by ACI code is larger than that by BS code for the same design load. This difference

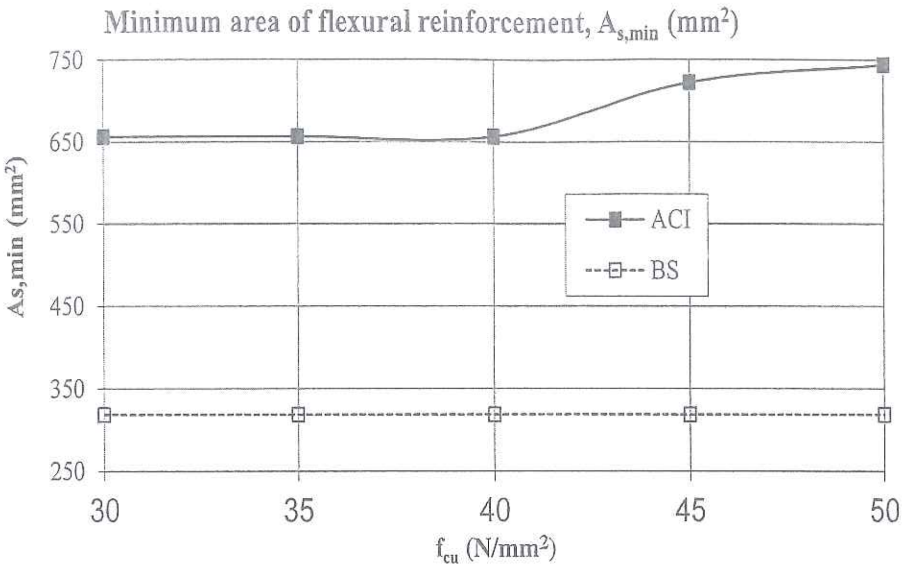

Fig. 7. Minimum area of flexural reinforcement with different f_{cu}

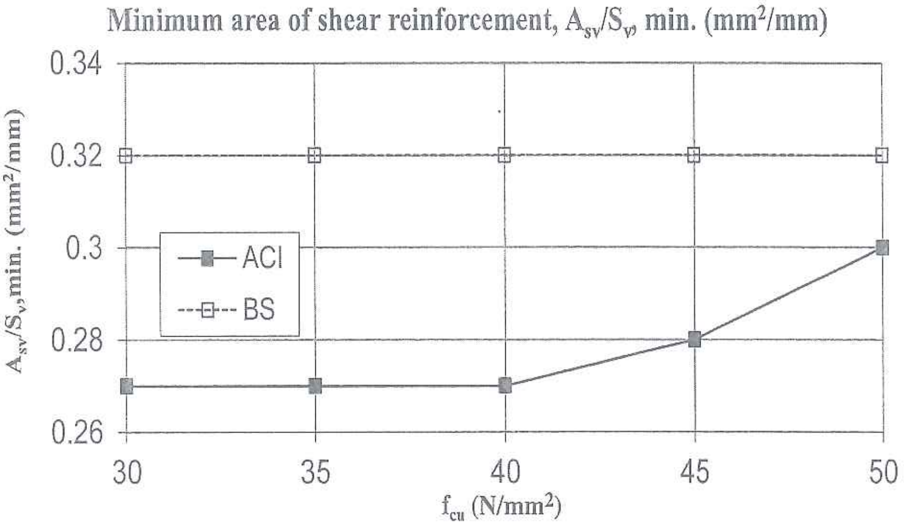

Fig. 8. Minimum area of shear reinforcement with different f_{cu}

becomes more pronounced with the increase of M_u/V_u ratio. It was found that empirical equations of shear capacity in BS and ACI codes have led to highly different results. It was also established that owing to differences in material safety factors, ACI equations lead to more required shear reinforcement than BS.

- The beam length that needs shear reinforcement (beyond which only minimum shear reinforcement is needed) required by BS code is shorter than that for ACI code. The difference becomes more pronounced with the increase of M_u/V_u ratio.
- The longitudinal and transverse torsional reinforcement required by ACI was found to be larger than that required by BS, and the difference in value between the reinforcement of the two codes is almost constant. It was found that these differences are due to differences in material safety factors.

Impact of Safety Factors on Ultimate Design Load

- The difference in the factor of safety for the dead load between the ACI and BS resulted in larger design bending moments and shear forces by the BS equations than the ACI ones. The diverging difference increases linearly with the increase of the dead load.
- For the resulting different design loads, it was found that both the longitudinal and transverse reinforcements required by the ACI are lower than the BS in all beams.

Punching Shear Strength (at Slab–Column Connection)

- For different column aspect ratios, the punching shear strength of flat slab–column connections calculated using the ACI code was found to be larger than that calculated using the BS code for the same geometry, materials, and loading conditions.
- In the ACI code, punching shear strength remains constant for different percentages of flexural reinforcement, whereas in the BS code, punching shear strength increases with increase of flexural reinforcement.
- For different slab thicknesses, ACI code estimates more punching shear strength than BS code.

Minimum Area of Flexural Reinforcement

Minimum area of flexural reinforcement required by ACI code is larger than BS code for RC rectangular beams.

Minimum Area of Shear Reinforcement

Minimum area of shear reinforcement required by ACI code is smaller than BS code for RC rectangular beams.

Recommendation

From the results of this research, it was found that the BS code requires less reinforcement than the ACI for the same design load. Contrarily, when the load safety factors are used in calculating the design loads from the service loads, the resulting factored loads using BS code are larger than the ACI code loads, which results in larger area of reinforcement by BS than the ACI. Hence, it is not easy to give preference of one code over the other for use in Oman and other countries that do not have national codes and allow both ACI and BS codes to be used. However, because SI units are becoming more and more enforced internationally, materials and references available in Oman and other Gulf states markets are conversant more toward SI units. To unify the knowledge of the design, municipality, and site engineers, it is recommended to use the BS code as a first choice until national codes are established. This will reduce the discrepancies between the design and construction phases in terms of standards, specifications, and materials. In the case that both ACI and BS codes unify the load safety factors while keeping the other design equations as they are now, the BS code will have preference over the ACI owing to fewer reinforcement requirements, which leads to cheaper construction.

Appendix I. Equations of Minimum Flexural Reinforcement in Beams

Situation	ACI 318:08 (Section 10.5)	BS 8110:97 (Table 3.25)
Flanged beams, web in tension		
$\frac{b_w}{b} < 0.4$	Larger of $\left(\frac{0.25\sqrt{f_c'}}{f_y} b_w d\right)$ or $\left(\frac{1.4}{f_y} b_w d\right)$	$0.0018 b_w h$
$\frac{b_w}{b} \geq 0.4$	Larger of $\left(\frac{0.25\sqrt{f_c'}}{f_y} b_w d\right)$ or $\left(\frac{1.4}{f_y} b_w d\right)$	$0.0013 b_w h$
Flanged beams, flange in tension		
T-beam	Larger of $\left(\frac{0.25\sqrt{f_c'}}{f_y} b_w d\right)$ or $\left(\frac{1.4}{f_y} b_w d\right)$	$0.0026 b_w h$
L-beam	Larger of $\left(\frac{0.25\sqrt{f_c'}}{f_y} b_w d\right)$ or $\left(\frac{1.4}{f_y} b_w d\right)$	$0.0020 b_w h$
Rectangular beams	Larger of $\left(\frac{0.25\sqrt{f_c'}}{f_y} b_w d\right)$ or $\left(\frac{1.4}{f_y} b_w d\right)$	$0.0013 b_w h$

Appendix II. Comparison of Formulas for Shear Reinforcement

ACI 318:08

$V_c = \phi V_n$ and $\phi = 0.75$ (for shear)

$$V_n = V_c + V_s$$

$$V_u = \phi(V_c + V_s) = \phi V_c + \phi V_s$$

$$V_s = \frac{V_u - \varphi V_c}{\phi}$$

$$\frac{A_{sv}}{s} = \frac{V_s}{d f_y} \text{ (ACI 318:08−11.4.7.2)}$$

$$\therefore \frac{A_{sv}}{s} = \frac{V_u - \phi V_c}{\phi d f_y}$$

Multiply the RHS with b/b

$$\therefore \frac{A_{sv}}{s} = \frac{V_u - \phi V_c}{\phi d f_y} \cdot \frac{b_w}{b_w}$$

$$\therefore \frac{A_{sv}}{s} = \frac{(v_u - \phi v_c) b_w}{\phi f_y}$$

where

$$v_c = \left(0.16\sqrt{f_c'} + 17\rho \frac{V_u d}{M_u}\right) \leq 0.29\sqrt{f_c'}$$

$$\therefore \frac{A_{sv}}{s} = \frac{(v_u - 0.75 v_c) b_w}{0.75 f_y}$$

$$\therefore s = 0.75 \frac{f_{yv} A_{sv}}{(v_u - 0.75 v_c) b_w}$$

BS 8110:97

$$\frac{A_{sv}}{s} = \frac{\left(v_u - \frac{v_c}{\gamma_{m,\text{ conc. in shear}}}\right) b_w}{\frac{f_y}{\gamma_{m,\text{ steel}}}}$$

where

$$v_c = \left(0.79 \left(\frac{100 A_s}{b_w d}\right)^{1/3} \left(\frac{400}{d}\right)^{1/4} \left(\frac{f_{cu}}{25}\right)^{1/3}\right)$$

$$\gamma_{m,\text{ conc. in shear}} = 1.25$$

$$\gamma_{m,\text{ steel}} = 1.05$$

$$\therefore \frac{A_{sv}}{s} = \frac{\left(v_u - \frac{v_c}{1.25}\right) b_w}{\frac{f_{yv}}{1.05}}$$

$$\therefore \frac{A_{sv}}{s} = \frac{(v_u - 0.8v_c)b_w}{0.95f_{yv}}$$

$$\therefore s = 0.95\frac{f_{yv}A_{sv}}{(v_u - 0.8v_c)b_w}$$

Notation

The following symbols are used in this paper:

A_{cp} = area enclosed by outside perimeter of concrete cross section;
A_l = area of longitudinal reinforcement to resist torsion;
$A_{l,\min}$ = minimum area of longitudinal reinforcement to resist torsion;
A_o = gross area enclosed by shear flow path;
A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement;
A_s = area of longitudinal tension reinforcement to resist bending moment;
$A_{s,\min}$ = minimum area of flexural reinforcement to resist bending moment;
A_{sv} = area of shear reinforcement to resist shear;
$A_{sv,t}$ = area of two legs of stirrups required for torsion;
A_t = area of one leg of a closed stirrup resisting torsion;
$A_{t,\min}$ = minimum area of shear reinforcement to resist torsion;
a = depth of equivalent rectangular stress block;
b = width of section flange;
b_w = width of section web;
d = effective depth of tension reinforcement (distance from extreme compression fiber to centroid of longitudinal tension reinforcement);
f_c' = characteristic cylinder compressive strength of concrete (150 mm × 300 mm);
f_{cu} = characteristic strength of concrete (150 × 150 × 150 mm concrete cube strength);
f_y = characteristic yield strength of longitudinal reinforcement for flexure;
f_{yl} = characteristic yield strength of longitudinal reinforcement for torsion;
f_{yv} = characteristic yield strength of transverse reinforcement;
h = overall depth of section;
$h_{\max}$ = larger dimension of rectangular cross section;
$h_{\min}$ = smaller dimension of rectangular cross section;
L = effective beam span;
M_u = ultimate flexural moment;
p_h = perimeter of centerline of outermost closed transverse torsional reinforcement;
S = center-to-center spacing of transverse reinforcement;
S_v = spacing of stirrups;
T_{cr} = torsional cracking moment;
T_n = nominal torsional moment strength;
T_u = ultimate design twisting moment;
V_c = nominal shear strength provided by concrete;
V_u = ultimate shear force;
v = design shear stress;
v_c = concrete shear strength;
v_t = torsional shear stress;
$v_{t,\min}$ = minimum torsional shear stress, above which reinforcement is required;
v_{tu} = maximum combined shear stress (shear plus torsion);
x_1 = smaller center to center dimension of rectangular stirrups;
y_1 = larger center to center dimension of rectangular stirrups;
Z = lever arm;
γ_m = partial safety factor for strength of material;
θ = angle between axis of strut, compression diagonal and tension chord of the member;
ρ = reinforcement ratio (A_s/bd); and
ϕ = strength reduction factor.

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